Field-of-Use Restrictions in Licensing Agreements*

Florian Schuett†

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Abstract

A widely used clause in license contracts – the field-of-use restriction (FOUR) – precludes licensees from operating outside of the technical field specified. When a technology has several distinct applications, FOUR allow the licensor to divide up his rights and attribute them to the lowest-cost producer in each field of use. This can improve production efficiency. With complex technologies, however, the boundaries of fields of use may be difficult to codify, entailing a risk of licensees’ rights overlapping. We explore how this affects the optimal license contract in a moral hazard framework where the licensor’s effort determines the probability of overlap. We show that depending on the contracting environment, the license agreement may include output restrictions and nonlinear royalty schemes.

Keywords: licensing, usage restrictions, overlap, royalties.
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† TILEC, CentER, Tilburg University, PO Box 90153, 5000 LE Tilburg, The Netherlands. Tel.: +31 13 466 4033. Fax: +31 13 466 3042. Email: f.schuett@uvt.nl.
1 Introduction

The licensing literature has by and large focused on license contracts covering the entire scope of a technology. In practice though, usage restrictions are pervasive. Since many technologies have several distinct applications, production efficiency often requires splitting usage rights among different firms. Field-of-use restrictions (henceforth, FOUR) are contractual provisions that enable the licensor to do precisely that: by precluding licensees from using the technology in fields other than those specified in the contract, they potentially allow the licensor to allocate production more efficiently among licensees. The lack of interest in the subject by economists can probably be attributed to their conception that splitting the rights to a technology is the same as licensing several unrelated technologies, and does not warrant special attention. Practitioners’ comments, however, suggest that with a complex technology, there is a risk that fields of use turn out to overlap. To realize the efficiency gains from FOUR, the licensor must exercise care in describing the boundaries between fields.

Licensing plays a key role both in determining incentives to innovate ex ante and in ensuring the diffusion of innovations ex post. Therefore, it is important to understand how firms design license contracts and how their payoffs and the feasibility of agreements depend on technology and industry characteristics. Several empirical studies on licensing have found usage restrictions to be common.\(^1\) This paper shows that FOUR, together with a risk of overlap between fields, can have profound implications for the design of the license contract.

The model presented below explores a particular situation where FOUR arise naturally.\(^2\) We look at the following simple setup. A licensor has a product innovation with two symmetric

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\(^1\) Caves, Crookell, and Killing (1983) survey 22 licensors and 34 licensees from the United States, Canada and the United Kingdom. They find that 34 percent of the license agreements in their sample include “market restrictions”, that is, restrictions preventing the licensee from selling outside certain specified markets (no distinction is made between territorial and field-of-use restrictions). Anand and Khanna (2000) conduct a large-scale inter-industry study of licensing behavior. Their sample consists of 1365 licensing deals involving at least one US corporation, as documented by the Securities Data Company. 37 percent of the agreements in their sample are identified as incorporating temporal, product or geographic restrictions, but again no distinction is made between the types of restriction. Anand and Khanna (2000) argue that this figure probably underestimates the actual frequency of such restrictions, because they are not always divulged in public announcements. Bessy and Brousseau (1998) compile a sample of 46 licensing agreements through a survey of French firms. They find that 87 percent of license contracts include some form of usage restriction. Contrary to the first two studies, they distinguish among the different types of restrictions. While geographical restrictions are the most common (58.7 percent), clauses limiting the field of application also turn out to be pervasive. They appear in 50 percent of the licences in the sample.

\(^2\) We should note that there may also be other motives for relying on usage restrictions than the one considered in the model. To give an example, the licensor may himself use the technology in one field, but may be unable or unwilling to exploit other applications. To take full advantage of his intellectual property, he may want to license other firms while at the same time protecting himself against competition in his own field of use. See Conti (2009) for the case of technology transfer from academic institutions to industry.
fields of use. There are two licensees, each of whom is specialized (i.e., has a cost advantage) in one of the fields. The licensor’s level of care in drafting the field-of-use clauses determines the probability of overlap, and writing a precise contract is costly. In the absence of overlap, both licensees are monopolists in their respective fields of use. In the event of overlap, each field of use becomes a duopoly where both licensees can produce. We assume that they compete in quantities, so that the equilibrium is asymmetric Cournot. Since the industry’s production technology is inefficient under duopoly (high-cost firms contribute to production in equilibrium), overlap reduces joint profits.

We consider three different contracting environments which differ in their degree of completeness. In the first, overlap is contractible. That is, it can be included as a contingency in the contract and courts can observe whether each licensee has exclusivity. This implies that royalty payments can depend on whether or not there is overlap. In the second, overlap is non-contractible, but the licensor’s effort – and thus the probability of overlap – is observable to the licensees. In the third, overlap is non-contractible and effort is unobservable.3,4

We derive the optimal contract in each of these environments without imposing any ad-hoc restriction on the royalty schemes permissible, except that they can depend only on each licensee’s total quantity. That is, we take a mechanism-design approach and maximize the licensor’s payoff over the set of feasible allocations. We restrict attention to a simple class of mechanisms: the licensor is limited to proposing a menu of contracts and cannot use any more sophisticated message game. From the optimal allocation, we deduce the properties of an optimal royalty scheme.

When overlap is contractible, the optimal license agreement consists of a fixed fee equal to the licensees’ profit in the case of overlap, as well as an output restriction that applies if there is overlap and a reward paid to the licensor if there is no overlap. The optimization problem can be broken down into two steps: first, the royalty scheme is chosen so as to maximize joint surplus given the realization of overlap. When there is no overlap, the optimal scheme is no royalty since royalties distort the licensees’ production decisions. When there is overlap, the royalty scheme is used to soften competition between licensees. Second, the reward is set so as to induce the licensor to exert the efficient level of effort. The reward that achieves this is

3 The environments we consider – overlap being perfectly contractible, or not at all – clearly are polar cases. In reality, courts may sometimes find breach of contract when fields of use overlap, and sometimes not. “Contractibility” would thus be probabilistic.

4 The environment where parties do not include overlap as a contingency could also be interpreted as a shortcut to account for the issues raised in Spier (1992), where contracts are strategically left incomplete by the principal in order to signal his type. In the current context, a clause specifying what happens in the event of overlap might be bad news about the licensor’s ability to separate fields of use.
equal to the difference between overlap and no-overlap profits.

When overlap is non-contractible but effort is observable, the royalty scheme can no longer be conditioned on the realization of overlap. The contract must now satisfy incentive compatibility constraints. We show that incentive compatibility requires that licensees produce more under duopoly (overlap) than under monopoly (no overlap). There is a conflict between incentive compatibility and efficiency (which requires that licensees produce less in the case of overlap) which is known as nonresponsiveness (Guesnerie and Laffont, 1984). Accordingly, a balance needs to be struck between softening competition in the case of duopoly and avoiding distortions in the case of monopoly. At the same time, there is no moral hazard problem, so that the royalty scheme can be designed with the sole objective of maximizing the expected profits of the industry. The optimal royalty scheme then reduces to an output restriction (quantity rationing). Costly effort to separate fields of use is always exerted.

When effort is unobservable, the royalty scheme has to fulfil an additional function: incentivizing the licensor to take appropriate care in drafting the field-of-use clauses. In fact, the patent holder has no incentive to exert effort unless his royalty income depends positively on effort. This means that effort is incompatible with the quantity rationing scheme that is optimal under observability, which necessarily leads to the same royalty payment regardless of overlap. Of course, the licensees will anticipate this. If they were offered such a scheme, they would be willing to pay only their expected profit given the minimum level of effort to obtain a license. Moreover, incentive compatibility implies that monopoly output cannot exceed duopoly output. Hence, a royalty scheme that induces effort must be such that output is strictly greater and the royalty payment strictly lower in the case of overlap. Solving for the optimal contract, we find that under some conditions, the resulting royalty scheme is no longer trivial and features royalties that are decreasing with output over some range. We identify a tradeoff between providing incentives to the licensor and producing at the efficient scale. While raising the duopoly quantity relaxes the incentive-compatibility constraint and thereby induces greater effort, it also moves duopoly output away from the efficient level. We show that this can be optimal when the cost of effort is sufficiently flat for small values of effort.

Our results are consistent with the observation by Taylor and Silberston (1973) and Kamien (1992) that royalty rates often decrease with output, at least when these royalty rates are interpreted as average rates. Since in the case of exclusivity, the royalty payment

\footnote{Kamien (1992, p. 346) states that "in the case of licensing the sale of a new product, the patentee often offers a lower royalty rate if sales exceed a certain prespecified level."}
is greater and the output lower than in the case of overlap, the model predicts that implied royalty rates per unit of output decrease with the quantity produced.\footnote{Alternative explanations for diminishing royalty rates have been proposed. The patentee may want to incentivize the licensee to push sales above those of other products (Kamien, 1992). Private information on the part of licensees concerning the value of the patented technology may also lead to the observed pattern (Beggs, 1992).}

**Related literature**

This paper is related to two separate strands of literature: the literature on licensing, particularly on the role of royalties in license agreements, and the literature on vertical restraints, particularly territorial restrictions. Much of the literature on licensing has been concerned with explaining the widespread use of royalties in practice, as documented by Taylor and Silberston (1973), Contractor (1981), and Rostoker (1984), which contrasts with the theoretical result that in a standard setup with risk-neutral firms and symmetric information, royalties are undesirable. This theoretical result was first established in the context of a (cost-reducing) process innovation (Kamien and Tauman, 1986).\footnote{Kamien and Tauman (1986) show that fixed fees dominate royalties for a patentee who licenses to a Cournot oligopoly. Katz and Shapiro (1986) obtain the additional result that auctioning off a fixed number of licenses, strictly below the total number of firms in the industry, does even better than a simple fixed fee. See Kamien (1992) for a survey of the literature on licensing of cost-reducing innovations.} It also holds in the context of product innovations: when the inventor is unable to work the patent himself, it is generally optimal to give exclusive rights to a single licensee.\footnote{It is sometimes argued that in the presence of increasing marginal costs of production it can be optimal to license several firms. This argument seems unconvincing. A single firm should be able to replicate what several firms are doing, for instance by setting up several production plants.} As we know from the literature on vertical control, due to the issue of double marginalization (Spengler, 1950), the licensor should then set royalties at zero (i.e., marginal cost) and extract the surplus through a fixed fee. Since the early 1990s, scholars have turned their attention to the conflict between theoretical predictions and empirical evidence. The explanations that have been put forward include adverse selection (Gallini and Wright, 1990; Beggs, 1992), moral hazard (Arora, 1995; Choi, 2001; Jensen and Thursby, 2001; Dechenaux, Thursby, and Thursby, 2008), and risk aversion (Bousquet, Cremer, Ivaldi, and Wolkowicz, 1998). This paper offers an alternative rationale for royalties based on the possibility that licensees’ fields of use may overlap, meaning that the licensor needs to be given incentives to take the appropriate level of care in drafting the license contract.

The vertical control literature has looked for conditions under which an upstream firm with market power will find it optimal to impose various kinds of vertical restraints on downstream
firms. Territorial restrictions are one such restraint, and several efficiency arguments for their adoption have been advanced. When retailers provide pre-sale services (such as advertising or consumer information) which have public-good aspects, competition will lead to free riding, a problem that can be solved by establishing local monopolies (Mathewson and Winter, 1984). When there is uncertainty about demand and/or cost parameters, and retailers are better informed than the manufacturer, exclusive territories make better use of the retailers’ information than retail competition since the latter drives market prices down to the wholesale price (Rey and Tirole, 1986). This literature generally assumes that downstream firms are identical (so that there is no reason in terms of cost efficiency for exclusive territories) and that defining (and enforcing) territorial restrictions is costless. Our approach is different in that we adopt a setup where there are natural advantages to exclusivity because there is a single most efficient firm in each field. Carelessness in the definition of fields may lead to overlapping rights. We investigate how the risk of overlap affects the desirability of field-of-use restrictions. While we focus on technical fields, our analysis may also apply to geographical territories in some cases: retailers with a lot of experience in selling in a particular geographical region may have advantages over competitors who lack knowledge of local characteristics, and territories may sometimes have to be appropriately defined to avoid ambiguity. A recent decision by the French supreme court is a case in point: overturning the decision of an appeals court, the judges ruled that a manufacturer who had granted an exclusive territory to a retailer but was also selling his products over the internet did not violate the contract.10

The remainder of this paper is organized as follows. Section 2 describes the setup of the model. Section 3 derives the optimal license contract in each of the three contracting environments we consider. Section 4 discusses the robustness of our results to alternative assumptions. Finally, Section 5 summarizes our results, highlights some empirical predictions, and briefly discusses implications for antitrust authorities.

2 A model of licensing with overlap between fields

Consider the following setup. A patent holder (P) wants to license his patented technology in two fields of use (1 and 2) where other firms have cost advantages.11 It is common knowledge

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9 See Katz (1989) and Rey and Vergé (2007) for an overview of the economics of vertical restraints.
11 We assume that the technology adds sufficient value to existing products to create a completely new market, rather than merely reducing costs. Much of the licensing literature has dealt with cost-reducing innovations, for which it is generally optimal to license many firms (at least for non-drastic innovations). For
that firm $L_1$ is the lowest-cost producer of application 1 and firm $L_2$ the lowest-cost producer of application 2. Letting $c_{ij}$ denote licensee $i$’s constant unit cost of producing application $j$, we have $c_{ii} < c_{ij}$ for $i = 1, 2$ and $j \neq i$. We normalize $c_{ii} = 0$ and assume $c_{ij} = c > 0$.

Licensees compete à la Cournot. The two fields of use are both equally profitable; inverse demand functions are identical and given by

$$p(Q_i) = \begin{cases} 1 - Q_i & \text{for } Q_i \leq 1 \\ 0 & \text{for } Q_i > 1, \end{cases}$$

where $Q_i$ is the total quantity sold in field $i$. Thus, letting $q_{ij}$ denote $L_i$’s output in field of use $j$, if $L_i$ sells $q_{ii}$ and $L_j$ sells $q_{ji}$, the price in market $i$ is $1 - q_{ii} - q_{ji}$.

Throughout the paper, we impose the following assumption on the cost $c$:

**Assumption 1.** The inefficient firm’s marginal cost satisfies $c < 1/2$.

In words, $c$ must be lower than the monopoly price that would prevail without royalties. This assumption ensures that the licensees can actually be a threat to each other. For some of our results, we use a slightly stronger assumption, whose significance is explained below:

**Assumption 2.** The inefficient firm’s marginal cost satisfies $c < 2/5$.

If it were costless to draft a contract that clearly separates the licensees’ fields of use, it would be optimal in this setup to give each licensee $L_i$, $i = 1, 2$, an exclusive license restricted to field $i$. This maximizes the parties’ joint surplus by allocating production entirely to the most efficient firm in each field. The parties can share the surplus through fixed upfront payments, creating proper incentives for the licensees by making them residual claimants. Royalties cause double marginalization and should therefore be avoided.

We will assume, however, that precise drafting is costly for the patent holder, and that the precision of the contractual language affects the probability of overlap. Denote the state of nature $s \in \{m, d\}$, where $m$ (monopoly) corresponds to exclusivity and $d$ (duopoly) corresponds to overlap. Let $e$ be the effort exerted by $P$ when writing the field-of-use clauses. Effort is chosen in the interval $[0, 1]$. With probability $e$, fields of use are well enough defined for each licensee to enjoy exclusivity ($s = m$). With probability $1 - e$, the fields of use are so broadly defined that they turn out to overlap ($s = d$). If there is overlap, each firm can produce in both fields of use. Thus, in particular, zero effort corresponds to the case of giving each firm a non-exclusive license for both fields of use. The cost of effort is $\psi(e)$, satisfying $\psi(e) = 0$ for $e \in [0, \underline{e}]$ and $\psi(e) > 0$ with $\psi' > 0$, $\psi'' > 0$ for $e \in (\underline{e}, 1]$, where $\underline{e} \in (0, 1)$. That the purposes of this paper, however, a framework where there are advantages to exclusivity is needed.
is, effort is costless up to some level $e$, above which it becomes increasingly costly. We also assume that the Inada conditions $\psi'(e) = 0$ and $\psi'(1) = \infty$ hold. In addition, for some of our results we will impose an assumption on the second and third derivatives of $\psi$ at $e$:

**Assumption 3.** The cost of effort satisfies $\psi''(e) = 0$ and $\psi'''(e) \leq [c(1-2e)]^2$.

The effort variable $e$ can be interpreted in several ways, with different implications for its observability by licensees. Assume there is a list of attributes on which fields could differ (this list could be large – possibly infinite, and include things such as size, color, and materials used). Only a few (possibly a single one) of them are relevant for cleanly defining the boundary between the two fields. Effort might consist of the time and money spent to find out the relevant attributes. In that case, effort is observable if licensees can find out which amount the licensor has invested, and unobservable otherwise. Alternatively, effort might be the number of attributes included in the contract, in a setting where adding a contingency to the contract is costly, as in Dye (1985) or Bajari and Tadelis (2001). In that case, effort is observable if there is a finite number of attributes, so that the number of attributes included in the contract is a sufficient statistic for effort, and unobservable if there is an infinity of attributes, rendering the number in the contract meaningless. In Section 3, we consider both the case where effort is observable and the case where it is unobservable.

Since we will investigate whether the use of a royalty can be beneficial when there is some probability of overlap, we have to make assumptions on the observability and verifiability of other variables. We start by clarifying our notion of overlap. When fields of use overlap, we maintain the assumption that the descriptions in the field-of-use clauses do not allow a court to establish whether a device produced under the terms of the license is destined for field 1 or 2. This seems natural since it is precisely the object of the contract to define what the fields of use are. Overlap corresponds to the case where the licensor has failed to draw a clear boundary. Thus, a court may be able to rule whether licenses overlap$^{12}$ – whether each licensee indeed has exclusivity in his field of use, or whether the licenses are so broad that each licensee can produce in both fields, thus violating exclusivity – but when the licenses turn out to overlap, the court generally cannot guess what the parties’ original intention was – which field to attribute exclusively to whom. Accordingly, the contract can depend only on the total output of each licensee (that is, $q_{ii} + q_{ij}$), and not on $q_{ij}$ individually.

We assume that the total output of each firm is observable and contractible. Effectively, this means we assume that courts can verify whether the terms of the license cover the

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$^{12}$ The possibility of using the event of overlap itself as a contingency is discussed in the following section.
products produced by the licensees, and thus – among other things – whether royalty payments are due. Prices are unobservable to the licensor. Royalties paid by $L_i$ cannot depend on $L_j$’s output.\(^{13}\) Apart from this, any royalty scheme (linear or nonlinear) that depends only on output is possible.

The timing is as follows (see Figure 1): at date 0, the contract is drafted by $P$ who chooses a level of care $e$. The contract specifies a fixed upfront payment $F$ and a royalty scheme $R : \mathbb{R}^+ \to \mathbb{R}$. That is, $R(q)$ is the royalty payment associated with an output of $q$ units of a product that uses the technology licensed. $L_1$ and $L_2$ each decide whether to accept or reject the contract.\(^{14}\) If a firm rejects it obtains zero profit regardless of the other firm’s decision. There are four possible outcomes: (accept, accept), (accept, reject), (reject, accept), and (reject, reject). We will look for conditions where both licensees accept the contract, which requires that each firm’s equilibrium payoff when accepting is non-negative. If the contracts are accepted, each pays the fixed fee to the patent holder. At date 1, the licensees learn whether their license permits them to enter the competitor’s field of use. Finally, at date 2, firms produce. If there is no overlap, they are monopolists. If there is overlap, they compete in both fields of use. The discount factor is equal to 1.

In the following section, we study the design of the license contract, taking as given that the licensor prefers licensing two firms to licensing a single firm supplying both markets.

\(^{13}\) This is equivalent to restricting the licensor to offering a menu of contracts, as we discuss below.

\(^{14}\) We assume that the contract offers are public. The literature on vertical control has sometimes used public contracts (see, e.g., Mathewson and Winter, 1984), and at other times secret contracts (see, e.g., Hart and Tirole, 1990). The issue that arises with secret offers in the context of licensing is one of commitment: while ex ante, the patent holder would like to commit himself, for instance, to license only one firm; once the contract is signed he is tempted to hand out additional licenses, eventually leading to a flooding of the market which erodes profits (Rey and Tirole, 2007). This is an example of contracting with externalities; see Segal (1999) for an excellent synthesis of many existing models which exhibit this feature. While an exclusivity clause may solve the problem in the particular example mentioned above, the issue is in fact much more general. In this paper, we abstract from the complexities that result from secret contract offers and focus on public offers.
The optimal design of the license contract

We will consider several scenarios concerning the verifiability of overlap and observability of effort. To establish a benchmark, we will derive the contract that would obtain if overlap were contractible.\textsuperscript{15} It is likely, though, that the event of overlap is not a perfectly contractible variable.\textsuperscript{16} Therefore, we will proceed to consider the other polar case where overlap is non-contractible. In a first step, we examine what the optimal contract is when licensees observe the amount of effort that \( P \) invests in the definition (and clean separation) of fields of use. In a second step, we investigate the opposite case where licensees do not observe \( e \), so that a moral hazard problem arises. To summarize, in what follows we study the optimal license contract in three different contracting environments: (1) when overlap is contractible, (2) when overlap is non-contractible and effort is observable, and (3) when overlap is non-contractible and effort is unobservable.

Throughout the paper, we will take a mechanism-design approach to the problem. That is, we look for allocations, consisting of pairs of quantities \( q(s) \) and associated royalty payments \( r(s) \) for each state of nature (overlap and exclusivity), which are uniquely implementable in Nash equilibrium. From this set of feasible allocations, we choose the one that maximizes the licensor’s expected profit.\textsuperscript{17} This allows us to characterize an optimal royalty scheme. Specifically, the licensees’ choices when facing an optimal royalty scheme must achieve the profit-maximizing allocation. Solving for this allocation thus gives us two precise points in the optimal royalty schedule and also speaks to the first derivative at those points.

3.1 Benchmark: contracting on overlap is possible

This section derives the first-best solution that obtains if the contract can be contingent on the state of nature \( s \) (i.e., on overlap). There is no issue of incentive compatibility restricting the set of implementable allocations in this case; we simply look for the quantities that maximize the parties’ joint payoffs. For completeness, we will consider both the case of observable effort and the case of unobservable effort. Note that because the parties are risk neutral, the efficient solution will be attained regardless of whether effort is observable.

\textsuperscript{15} If it may seem implausible that real-world contracts would actually be contingent on overlap, it is interesting to note that they sometimes are: for an example of a license agreement which includes an overlap clause from the SEC info database, see http://www.secinfo.com/dRqWm.8vWQA.htm (accessed on May 11, 2011).
\textsuperscript{16} In practice, \( P \) can perhaps be held responsible for careless drafting (overlap amounts to a breach of contract since licensees do not get the exclusivity they signed up for), but whether a court will actually find him liable is uncertain at best.
\textsuperscript{17} As stated in the introduction and explained in more detail below, there is one qualification: we will restrict the available mechanisms to the simple class of incentive-compatible contracts.
With contractible overlap and observable effort, the licensor’s problem is

\[
\max_{e; \{F_i; (q_i(m), r_i(m)); (q_i(d), r_i(d))\}_{i=1,2}} \sum_i F_i + e \sum_i r_i(m) + (1 - e) \sum_i r_i(d) - \psi(e),
\]

subject to \( F_i \leq e[\Pi_m(q_i(m)) - r_i(m)] + (1 - e)[\Pi_d(q_i(d), q_j(d)) - r_i(d)], \quad i = 1, 2; \ j \neq i, \)

where \( \Pi_s \) is the equilibrium (gross of royalty) profit made by a licensee given the market structure, \( s \), and each firm’s total output (this will be defined more precisely below). Notice that, consistent with the assumption that the royalty scheme cannot depend on \( q_{ij} \) in the case of overlap, the mechanism only specifies a total quantity \( q_i(d) = q_{ii} + q_{ij} \).

Since the licensor makes a take-it-or-leave-it offer, he will choose \( F \) as large as possible; hence, the constraint must be binding at the optimum. The problem becomes

\[
\max_{e; \{q_i(m), r_i(m); q_i(d), r_i(d)\}_{i=1,2}} \ e \sum_i \Pi_m(q_i(m)) + (1 - e) \sum_i \Pi_d(q_i(d), q_j(d)) - \psi(e). \tag{1}
\]

The choice of quantities is independent of \( e \), so the problem of finding the optimal license contract can be broken down into two steps: first, determining the optimal quantity for any given realization of overlap, and deducing the royalty scheme that achieves it (note that since the event of overlap can be included as a contingency in the contract, the royalty scheme can depend on its realization so that there can be two different royalty schemes \( R_m \) and \( R_d \)); second, finding the level of effort that maximizes the expected joint profit given the difference between licensee profits in the overlap and no-overlap cases. Note also that, since \( P \) can extract the licensees’ surplus through the fixed fee, the use of a royalty will be motivated solely by the desire to influence the licensees’ decisions (as opposed to the extraction of surplus).

**No overlap: monopoly**

When both fields of use are cleanly separated (\( s = m \)), each licensee is a monopolist in his market. Each licensee’s monopoly profit as a function of quantity \( q \) is

\[
\Pi_m(q) = [1 - q]q. \tag{2}
\]

The profit-maximizing quantity is \( q^*_m = 1/2 \). It is obvious from (1) that the mechanism should implement \( q_i(m) = q^*_m \ \forall i \).

Thus, any royalty scheme that induces the licensee to produce the monopoly quantity \( q^*_m \) is a solution. One particular solution that stands out for its simplicity is no royalty at all, \( R_m(q) = 0 \ \forall q \). This is the standard double marginalization argument (Spengler, 1950):
in a vertical relationship, the upstream firm should charge a price to the downstream firm that equals the marginal cost of the input supplied, and extract the surplus through a fixed upfront payment. Here, the input is simply information (technological knowledge), which has zero marginal cost.

**Overlap: duopoly**

When fields of use overlap \((s = d)\), both licenses are effectively non-exclusive: licensee \(L_i\) faces competition in field \(i\) from (less efficient) licensee \(L_j\) and vice versa. While in the case of monopoly, royalties are undesirable because they create a distortion, in the duopoly case royalties can be useful as a means of softening competition. We proceed as follows: first, we derive the licensees’ equilibrium behavior given that each of them is required to produce a total quantity \(q_i(d)\); second, we determine the quantity that maximizes industry profit subject to the licensees’ optimizing behavior; third, we deduce the royalty scheme that achieves those quantities.

Since, by assumption, firms compete in quantities, \(L_i\) maximizes over \(q_{ii}\) and \(q_{ij}\):

\[
[1 - q_{ii} - q_{ji}]q_{ii} + [1 - q_{jj} - q_{ij} - c]q_{ij},
\]

subject to \(q_{ii} + q_{ij} = q_i(d)\), while taking \(q_{ji}\) and \(q_{jj}\) as given. The following lemma characterizes the equilibrium values of \(q_{ii}\) and \(q_{ij}\).

**Lemma 1.** Let \(q_i(d)\) be the total quantity to be produced by licensee \(i\) in the case of overlap. If \(q_i(d) \geq c\) and \(q_j(d) \geq c\), the Cournot-Nash equilibrium of the game is

\[
q_{ii} = \frac{q_i(d) + c}{2},
q_{ij} = \frac{q_i(d) - c}{2}
\]

for all \(i, j \neq i\) and for any \(q_j(d) \geq c\).

If \(q_i(d) \geq c\) and \(q_j(d) < c\), the equilibrium is

\[
q_{ii} = \frac{2q_i(d) + q_j(d) + c}{4},
q_{ij} = \frac{2q_i(d) - q_j(d) - c}{4},
q_{jj} = q_j(d),
q_{ji} = 0.
\]
Finally, if \( q_i(d) < c \) and \( q_j(d) < c \), the equilibrium is

\[
q_{ii} = q_i(d) \\
q_{ij} = 0
\]

for all \( i, j \neq i \) and for any \( 0 \leq q_j(d) < c \).

**Proof:** Rewriting (3) using \( q_i(d) \) in the constraint and taking into account that \( L_j \) faces an analogous problem so that \( q_{ji} = q_j(d) - q_{jj} \), \( L_i \)'s problem becomes

\[
\max_{q_{ii}} [1 - q_{ii} - (q_j(d) - q_{jj})]q_{ii} + [1 - q_{jj} - (q_i(d) - q_{ii}) - c](q_i(d) - q_{ii}).
\]

The first-order condition of the problem is

\[
1 - 2q_{ii} - q_j(d) + q_{jj} = 1 - q_{jj} - 2(q_i(d) - q_{ii}) - c.
\]

Solving for \( q_{ii} \) and replacing \( q_{jj} \) from the first-order condition of \( L_j \)'s problem, we obtain

\[
q_{ii} = \frac{c - q_i(d) + 2q_i(d) + [c - q_i(d) + 2q_{ii} + 2q_j(d)]/2}{4},
\]

which can be solved for the equilibrium \( q_{ii} \), which, in turn, can be replaced in the constraint to obtain \( q_{ij} \), yielding the expressions claimed in the first part of the lemma (for the case where \( q_i(d) > c \) for all \( i \)). These are valid solutions as long as \( q_{ij} \geq 0 \) for all \( i \), which is the case as long as \( q_i(d) \geq c \). When \( q_i(d) < c \), \( L_i \)'s output in field \( j \), \( q_{ij} \), is zero, while \( q_{ii} = q_i(d) \).

Modifying the first-order condition accordingly yields the second part of the lemma. ■

The difficulty for the licensor is that he can only control the total quantity produced by each licensee. He has no influence on the distribution of output between the two fields of use – except when the total quantity is so restrictive as to deter a licensee from entering the competitor’s market altogether. In all other cases, the Cournot-Nash equilibrium has each licensee producing in both fields of use, with the share of the more efficient firm determined by the difference in marginal costs. As a result, in equilibrium some of the units sold are generally contributed by high-cost producers, making the industry’s production technology inefficient. The licensor will optimally react to this by inducing less than monopoly output.

Note also that Lemma 1 implies that it can never be optimal for the licensor to choose a quantity \( q_i(d) \) that is strictly below \( c \). Any total output greater than \( 2c \) can be produced more efficiently if \( q_i(d) \geq c \) for all \( i \), because a larger proportion is then produced by the efficient firm. Replacing the quantities from the first part of the lemma in (3), we obtain:
Corollary. The equilibrium profit of licensee \( i \) as a function of total quantities \( q_i(d) \) and \( q_j(d) \) when \( q_i(d) \geq c \) for all \( i \) is:

\[
\Pi_d(q_i(d), q_j(d)) = \left[ 1 - \frac{q_i(d) + q_j(d) - c}{2} \right] q_i(d) + \frac{c^2}{2}.
\] (5)

All that matters for industry profits is aggregate output, \( q_i(d) + q_j(d) \), and not how it is shared between the licensees. Therefore, from now on we assume that the licensor treats both licensees symmetrically, so that \( q_i(d) = q_j(d) = q(d) \), which greatly simplifies notation. Let \( \hat{\Pi}_d(q) \) denote the duopoly profit with symmetric total quantities. According to Lemma 1, we have \( \hat{\Pi}_d(q) = \Pi_m(q) \) for \( q \leq c \) and \( \hat{\Pi}_d(q) = \Pi_d(q, q) = \left[ 1 - q - c/2 \right] q + c^2/2 < \Pi_m(q) \) for \( q > c \). The licensor’s problem is to choose \( q(d) \) so as to

\[
\max_{q(d)} \hat{\Pi}_d(q(d)) = \left[ 1 - q(d) \right] q(d) - c \cdot \max\{0, (q(d) - c)/2\}.
\] (6)

The next lemma describes the solution to this problem.

Lemma 2. If \( c \leq 2/5 \), the quantity \( q_d^* \) that maximizes industry profits is given by

\[
q_d^* = \frac{1 - c/2}{2}
\]

and both firms contribute to production with \( q_{ii} = (2 + 3c)/8 \) and \( q_{ij} = (2 - 5c)/8 \) for all \( i \).

If \( c > 2/5 \), only the low-cost producer is active, i.e., \( q_{ij} = 0 \), and \( q_d^* = c \). In both cases, the aggregate output is lower than the monopoly quantity: \( q_d^* < q_m^* \).

Proof: The optimization program

\[
\max_q \left[ 1 - q \right] q - \frac{c}{2} (q - c)
\]

leads to the first-order condition \( 1 - 2q - c/2 = 0 \), the solution of which is \( q_d^* = \frac{1 - c/2}{2} \). Given that \( \hat{\Pi}_d \) is strictly concave (\( \hat{\Pi}_d'' = -2 < 0 \)), this is a valid solution to (6) as long as it is greater than \( c \), that is

\[
\frac{1 - c/2}{2} \geq c \Leftrightarrow c \leq 2/5.
\]

One can then determine \( q_{ii} \) and \( q_{ij} \) using Lemma 1. If \( c > 2/5 \), we have a corner solution so that the second part of Lemma 2 applies. As for the last claim, \( \frac{1 - c/2}{2} < \frac{1}{2} \) since \( c > 0 \), while \( c < \frac{1}{2} \) by Assumption 1. ■

In order to limit the damage caused by overlap, there are two possibilities. One can shut down the inefficient firm by reducing output to \( c \). Alternatively, one can be less restrictive,
at the expense of involving the inefficient firm in production. The intuition for Lemma 2 is that, when \( c \) is low, one needs to sacrifice much output to deter each firm from entering its competitor’s market, while at the same time the efficiency loss from involving it in production is not too important. Therefore, it is optimal to allow both firms to be active, albeit at an aggregate level of activity that is below \( q^*_m \). By contrast, when \( c \) is high, having both firms contribute to production is very inefficient, and at the same time deterrence is not too costly. Therefore, restricting output to \( c \) is optimal. Figure 2 illustrates the case where \( c < 2/5 \), so that letting both firms produce is optimal. In the figure, \( q^{\text{Cournot}}_d \) denotes the sum of the two firms’ outputs in the Cournot equilibrium that arises absent royalties.\(^{18}\)

There are many royalty schemes that allow the licensor to implement \( q(d) = q^*_d \). The simplest is a quantity restriction that limits output to \( q^*_d \), that is,

\[
R_d(q) = \begin{cases} 
0 & \text{for } q \leq q^*_d \\
\infty & \text{for } q > q^*_d.
\end{cases}
\]

He can also use a unit royalty, \( R_d(q) = \rho q \) with

\[
\rho = \begin{cases} 
\frac{1-c/2}{4} & \text{for } c < 2/5 \\
1 - 2c & \text{for } c \geq 2/5.
\end{cases}
\]

**The licensor’s effort choice**

When effort is observable, the licensor chooses his level of care so as to maximize his expected profits. In the preceding analysis, we have derived the optimal quantity for each

---

\(^{18}\) The Cournot quantity absent royalties is \( q^{\text{Cournot}}_d = (2 - c)/3 > 1/2 = q^*_m \) for any \( c < 1/2 \).
realization of overlap; let the associated levels of (gross of royalty) profit be \( \pi_m^* \equiv [1 - q_m^*]q_m^* \) and \( \pi_d^* \equiv [1 - q_d^* - c/2]q_d^* + c^2/2 \). The licensor’s choice of \( e \) then simply maximizes

\[
e\pi_m^* + (1 - e)\pi_d^* - \psi(e)/2.
\]  

(7)

When effort is unobservable, the licensor needs to be given incentives to choose the efficient level of effort. This can be achieved by rewarding the licensor for avoiding overlap, i.e., through a reward clause in the contract that stipulates a higher royalty payment when there is no overlap than when there is. Proposition 1 summarizes the features of the optimal contract when the realization of overlap is verifiable by a court of justice. (We have \( r_i(s) = r_j(s) = r(s) \) and \( F_i = F_j = F \) because of the symmetric treatment of licensees.)

**Proposition 1.** Suppose overlap is contractible. The optimal contract \((e^c, F^c, (q^c(m), r^c(m)), (q^c(d), r^c(d)))\) then takes the following form:

(i) The level of care exerted by \( P \) is \( e^c \), determined by \( \psi'(e^c) = 2(\pi_m^* - \pi_d^*) \);

(ii) the fixed fee is \( F^c = \pi_d^* \);

(iii) when there is no overlap, output is \( q^c(m) = q_m^* \) and \( r^c(m) = \pi_m^* - \pi_d^* \);

(iv) when there is overlap, output is \( q^c(d) = q_d^* \) and \( r^c(d) = 0 \).

This can be implemented through a royalty scheme \( R^c_m(q) = \pi_m^* - \pi_d^* \) in the case of exclusivity and

\[
R^c_d(q) = \begin{cases} 
0 & \text{for } q \leq q_d^* \\
\infty & \text{for } q > q_d^* 
\end{cases}
\]

for the case of overlap.

**Proof:** The previous analysis has shown that, in the no-overlap case, the royalty scheme must be non distortive (which is the case here because \( R^c_m \) does not depend on output), while in the case of overlap, Lemma 2 has shown that an optimal royalty scheme must induce \( q_d^* \) (which can be achieved through quantity rationing, as in the case of \( R^c_d \), because \( q_d^* \) is strictly smaller than the equilibrium output the licensees would choose in the absence of an output

\[
\begin{align*}
\pi_m^* &= \frac{1}{4} \\
\pi_d^* &= \begin{cases} 
\frac{1}{4}[1 - c + 9c^2/4] & \text{for } c < 2/5 \\ 
c(1 - c) & \text{for } c \geq 2/5.
\end{cases}
\end{align*}
\]
restriction, given by $q_d^{Cournot} = (2 - c)/3$. When effort is observable, maximizing (7) leads to the first-order condition

$$\pi^*_m - \pi^*_d = \psi'(e)/2.$$  

(8)

The assumptions on $\psi$ ensure that this condition is necessary as well as sufficient. Each licensee’s expected profit then is

$$e^c(\pi^*_m - r^c(m)) + (1 - e^c)(\pi^*_d - r^c(d)) = e^c(\pi^*_m - (\pi^*_m - \pi^*_d)) + (1 - e^c)\pi^*_d = \pi^*_d,$$

which $P$ extracts through the fixed fee $F^c$. His total payoff, given by the fixed fee plus expected royalty revenue less the cost of effort, is $2(\pi^*_d + e^c(\pi^*_m - \pi^*_d)) - \psi(e^c)$, i.e., the entire aggregate surplus of the relationship.

Turning to the case where effort is unobservable, we have to show that, given the royalty payments $r^c(m)$ and $r^c(d)$, the licensor wants to choose $e^c$. His preferred level of effort is obtained by solving

$$\max_e e^c r^c(m) + (1 - e) r^c(d) - \psi(e)/2 = e(\pi^*_m - \pi^*_d) - \psi(e)/2,$$

the first-order condition of which coincides with (8). ■

Proposition 1 says that the optimal level of effort is such that the marginal cost of effort equals the marginal benefit of effort, the latter being given by twice (because there are two fields of use) the difference between monopoly and duopoly profits. The fixed fee is equal to the duopoly profit. When there is no overlap, the licensor is rewarded through a second fixed payment that does not affect the licensees’ choice of quantities but gives him incentives to take the appropriate care.

There are several reasons why licensing contracts may not include an overlap clause. The licensor may be reluctant to insert such a clause because it may be bad news about his ability to cleanly separate fields of use. That is, he may leave the contract incomplete for signaling purposes, as in Spier (1992). In this model, we have assumed no information asymmetry with respect to the licensor’s cost of effort and instead pursue an alternative route. Overlap may be difficult to verify for a court of justice, or even for the licensor himself: to establish overlap, what needs to be proved is that a good produced by licensee $i$ within the terms of his license competes with one of licensee $j$’s products. This can be less than straightforward, especially when the licensees have private information on market prices, as we have assumed.\footnote{In a sense, the idea is that courts are unaware of consumers’ preferences: they cannot determine whether two products are substitutes, complements or unrelated.} Moreover, as argued by Cestone and White (2003), when enforcing certain
clauses is expensive and highly uncertain, contracting parties may want to complement legal incentives with financial incentives. For these reasons, the following sections consider the case where the license contract cannot be contingent on overlap.

For the remainder of the analysis, we restrict attention to the case where it is optimal to involve both licensees in production when there is overlap, i.e., we impose Assumption 2. This makes the analysis less cumbersome by removing the necessity of considering different cases and reducing the incidence of corner solutions. It does not obscure any important insights.

3.2 Overlap non-contractible but effort observable

We now drop the assumption that overlap can be included as a contingency in the contract. When fields of use overlap and $L_i$ sells a product in field $j$, a court is unable to hold the licensor responsible for the lack of exclusivity enjoyed by $L_j$. In the absence of overlap, there is no problem because the terms of the license do not allow $L_i$ to launch a product that competes with $L_j$’s.

Overlap being nonverifiable, the mechanism-design problem is no longer as straightforward. Nevertheless, if we assume that all three contracting parties learn the state of the world before the production stage, the fundamental result by Maskin (1977) tells us that the first-best allocation derived in Proposition 1 is Nash implementable because it satisfies monotonicity. And even if we assume that only the licensees learn the state of the world, it is still possible to implement the first best through a direct revelation mechanism where the allocation depends on both players’ messages, as we show in Appendix C. Here, we will keep the problem interesting by restricting the set of mechanisms available to the licensor. Specifically, we assume that the licensor can ask the licensees to report the state of the world, but that each licensee’s output and royalty payment can only depend on his own report, and not on the other licensee’s message. (Nor can any other sophisticated message game be played.) This is equivalent to having the licensor offer each licensee a menu of contracts. By doing this, we meet a much-voiced concern with implementation theory according to which mechanisms are often excessively complex.

Formally, we must now add incentive-compatibility constraints to the problem to ensure

\footnote{We still assume the courts to be able to ascertain whether a product falls within the terms of the license.}

\footnote{See, e.g., Dewatripont (1992), who notes that “the search for positive implementation results has led to a series of excessively sophisticated games. While there is no reason for \textit{a priori} restricting the set of acceptable mechanisms, it is of some concern that the presumed outcomes of these games rely on extremely subtle equilibrium behavior. If one were to actually apply these games in practice, one can doubt that the agents would play the equilibrium strategies.” Dewatripont therefore recommends constraining the set of possible games from the start.}
that it is in the licensees’ interest to choose the contract corresponding to the underlying 
state of the world. In addition, we have to make sure that the equilibrium is unique, i.e., that 
there is no non-truthful equilibrium. The licensor’s problem becomes 

$$\max_{e:F(q(m),r(m)):(q(d),r(d))} F + e r(m) + (1 - e) r(d) - \psi(e)/2,$$

subject to

$$F \leq e \left[ \Pi_m(q(m)) - r(m) \right] + (1 - e) \left[ \Pi_d(q(d), q(d)) - r(d) \right]$$

(9)

$$\Pi_m(q(m)) - r(m) \geq \Pi_m(q(d)) - r(d)$$

(10)

$$\Pi_d(q(d), q(d)) - r(d) \geq \Pi_d(q(m), q(d)) - r(m)$$

(11)

$$\Pi_d(q(d), q(m)) - r(d) \geq \Pi_d(q(m), q(m)) - r(m).$$

(12)

In words, the licensees must prefer to report $m$ when they have exclusivity, and each licensee 
must prefer to report $d$ regardless of the other’s report when there is overlap. Constraint (10) 
states that when the state of the world is $m$, each licensee must be better off producing $q(m)$ 
and paying $r(m)$ than producing $q(d)$ and paying $r(d)$. Constraint (11) states that when the 
state of the world is $d$, each licensee must be better off producing $q(d)$ and paying $r(d)$ than 
producing $q(m)$ and paying $r(m)$ conditional on the other licensee choosing $q(d)$. Constraint 
(12) makes the same statement as (11) conditional on the other licensee choosing $q(m)$; thus, 
it guarantees uniqueness.

As the following lemma shows, the incentive-compatibility constraints (10) and (11) 
severely restrict the set of implementable allocations.

**Lemma 3.** When overlap is non-verifiable, a necessary and sufficient condition for any pair 
of outputs $(q(m) \geq c, q(d) \geq c)$ to be implementable is

$$q(d) \geq q(m).$$

**Proof:** Adding up (10) and (11) and rearranging, we have

$$
\Pi_m(q(m)) - \Pi_m(q(d)) \geq \Pi_d(q(m), q(d)) - \Pi_d(q(d), q(d)).
$$

Substituting from (2) and (5), this becomes

$$
[1 - q(m)]q(m) - [1 - q(d)]q(d) \geq \left[ 1 - \frac{q(m) + q(d)}{2} - \frac{c}{2} \right] q(m)
- \left[ 1 - q(d) - \frac{c}{2} \right] q(d),
$$

19
which, after simplification, yields

$$(q(d) - q(m))(q(m) - c) \geq 0.$$  

As for the sufficiency part, we now show that (12) is implied by (11) when the monotonicity condition, $q(d) \geq q(m)$, holds. Constraint (12) is satisfied whenever (11) is if

$$\Pi_d(q(d), q(m)) - \Pi_d(q(d), q(d)) \geq \Pi_d(q(m), q(m)) - \Pi_d(q(m), q(d)).$$

This is true if

$$\frac{\partial}{\partial q_i} \left[ \Pi_d(q_i, q_j) - \Pi_d(q_i, q'_j) \right] > 0,$$

$$\iff 1 - q_i - \frac{q_j}{2} - \frac{c}{2} > 1 - q_i - \frac{q'_j}{2} - \frac{c}{2}$$

which is the case for any $q_j < q'_j$. ■

The intuition for this result is related to the fact that when the firms are competing with each other they do not fully internalize the effect of an increase in output on the market price. Each firm only takes into account the effect on the price of those units of output the firms sells itself, but not the effect on the price of the units sold by its competitor. Here, this implies that the firms always have a stronger incentive to increase output when there is overlap than when there is no overlap. As a result, a contract asking firms to produce more when they have exclusivity than when there is overlap is not incentive compatible: in the event of overlap, firms will claim to have exclusivity in order to produce a higher quantity, and there exists no royalty payment that would induce them to truthfully report overlap without also inducing them to report overlap when in fact they have exclusivity.

Lemma 3 means the licensor faces a phenomenon of nonresponsiveness (Guesnerie and Laffont, 1984): while efficiency requires $q(d) < q(m)$ (recall Lemma 2), he can only implement allocations satisfying $q(d) \geq q(m)$. Proposition 2 characterizes the optimal contract.

**Proposition 2.** Suppose overlap is non-contractible, effort is observable, and Assumption 2 holds. Then, the optimal license contract $(e^o, F^o, (q^o(m), r^o(m)), (q^o(d), r^o(d)))$ has the following properties:

(i) Output is the same irrespective of overlap: $q^o(m) = q^o(d) = \bar{q}^o$;

(ii) $\bar{q}^o$ and effort $e^o$ solve

$$\psi'(e^o) = c(\bar{q}^o - c)$$  \hspace{1cm} (13)
$$\bar{q}^o = \frac{1 - (1 - e^o)c/2}{2};$$  \hspace{1cm} (14)
(iii) the fixed fee is \( F^o = [1 - \bar{q}^o]q^o - (1 - e^o)\frac{c(\bar{q}^o - c)}{2}; \)

(iv) \( r^o(m) = r^o(d) = 0. \)

This can be implemented through a royalty scheme that takes the form of an output restriction, that is,

\[
R(q) = \begin{cases} 
0 & \text{for } q \leq \bar{q}^o \\
\infty & \text{for } q > \bar{q}^o.
\end{cases}
\]

**Proof:** Using Lemma 3 and the fact that the ex ante participation constraint (9) must bind, the licensor’s optimization program is

\[
\max_{e,q} e\Pi_m(q(m)) + (1 - e)\hat{\Pi}_d(q(d)) - \frac{\psi(e)}{2}
\]

subject to \( q(m) \leq q(d). \) Let \( \lambda \) denote the multiplier associated with the constraint. Optimizing with respect to \( q(m) \) and \( q(d) \) yields

\[
\begin{align*}
\epsilon\Pi_m'(q(m)) &= \lambda \\
(1 - e)\Pi_d'(q(d)) &= -\lambda.
\end{align*}
\]

We show first that the constraint must be binding. \( \Pi'_m \) and \( \Pi'_d \) cannot be simultaneously zero for any pair of outputs such that \( q(m) \leq q(d). \) By the Inada conditions, \( e = 1 \) cannot be optimal. Suppose that \( e = 0 \) and \( \lambda = 0. \) Then, \( q(d) = q_d^* \) by (17) and \( q(m) < q_d^*. \) But this cannot be optimal since the licensor could strictly increase his payoff by setting \( e = \epsilon \) and \( q(m) = q_d^* \) instead (because \( \Pi_m(q_d^*) > \hat{\Pi}_d(q_d^*) \)). Thus, \( \lambda > 0. \)

Letting \( q(m) = q(d) \equiv \bar{q}, \) the problem can be simplified to

\[
\max_{\epsilon, \bar{q}} \epsilon\Pi_m(\bar{q}) + (1 - e)\hat{\Pi}_d(\bar{q}) - \frac{\psi(e)}{2}.
\]

The first-order conditions for an interior solution are

\[
\begin{align*}
\Pi_m(\bar{q}) - \hat{\Pi}_d(\bar{q}) &= \frac{\psi'(e)}{2} \\
\epsilon\Pi_m'(\bar{q}) + (1 - e)\hat{\Pi}_d'(\bar{q}) &= 0,
\end{align*}
\]

which can be simplified to the claimed expressions determining \( e^o \) and \( \bar{q}^o, \) (13) and (14).

What remains to be shown is that the solution will indeed be interior. From inspection of (18), it is clear that \( \bar{q}^o \) must be in \([q_d^*, q_m^*].\) By Assumption 2, \( \Pi_m(q) > \hat{\Pi}_d(q) \) for any \( q \in [q_d^*, q_m^*]. \) It follows that \( e^o > \epsilon. \) Suppose otherwise, i.e., \( 0 \leq e^o \leq \epsilon. \) Increasing \( e \) slightly generates only second-order losses (since \( \psi(e) = 0 \) for \( 0 \leq e \leq \epsilon \) and \( \psi'(\epsilon) = 0 \)) and first-order
gains, so \( e \leq e' \) cannot be optimal. Similarly, \( e = 1 \) cannot be optimal since \( \psi'(1) = \infty \). We conclude that \( e < e' < 1 \). But given any interior \( e \), \( e' \) must be interior as well. Suppose \( \tilde{q}^* \) were equal to \( q_d^* \) or \( q_m^* \) instead. Moving away slightly to the interior again causes only second-order losses (from the definition of \( q_d^* \) and \( q_m^* \)) and first-order gains (since \( 0 < e < 1 \)). Thus, the solution must be interior.

The second-order conditions for a local maximum are

\[
- \frac{\psi''(e)}{2} \leq 0 \tag{19}
\]

\[
- \frac{\psi''(e)}{2} [e \Pi'_m(\tilde{q}) + (1 - e) \hat{\Pi}'_d(\tilde{q})] \geq [\Pi'_m(\tilde{q}) - \hat{\Pi}'_d(\tilde{q})]^2. \tag{20}
\]

Condition (19) holds by assumption. Noting that \( \Pi'_m = \hat{\Pi}'_d = -2 \) and \( \Pi'_m - \hat{\Pi}'_d = c/2 \), condition (20) is equivalent to

\[
\psi''(e) > \frac{c^2}{4}.
\]

Hence, the solution will be characterized by the first-order conditions (13) and (14) as well as the second-order condition \( \psi''(e^*) \geq c^2/4 \). ■

Proposition 2 shows that when overlap is non-contractible but effort is observable, we get a pooling allocation, i.e., \( q(m) = q(d) = \tilde{q} \). This is not surprising since nonresponsiveness is frequently associated with pooling of types. How should the quantity on which to pool be chosen? The optimal level of effort and \( \tilde{q} \) are interdependent: the lower the probability of overlap, the more the licensees’ production should approach \( q_m^* \). The higher the production, the higher is the difference between monopoly and duopoly profits (see Figure 2), and the stronger are the incentives to avoid overlap.

One of the implications of Proposition 2 is that \( e < e' < 1 \), which means that, in spite of the inefficiency caused by nonresponsiveness, field-of-use restrictions are still preferred to non-exclusive licenses. Inducing the licensees to produce the optimal quantity can most easily be achieved through an output restriction. For other distortive royalty schemes, monopoly output would be reduced to a level strictly below duopoly output, which is undesirable. A constant per-unit royalty (\( R(q) = \rho q \)), for example, cannot achieve the allocation resulting from the contract described in Proposition 2. Such a royalty scheme leads to quantities \( q^r(m) = (1 - \rho)/2 \) and \( q^r(d) = (2 - 2\rho - c)/3 \). Thus, \( q^r(m) \leq q^r(d) \) as long as \( \rho \leq 1 - 2c \). This condition must always hold; as can be shown, larger values of \( \rho \) can never be optimal.\(^{23}\)

\(^{23}\) If \( s = m \), firm \( i \) is a monopolist with effective marginal cost \( \rho \). The profit-maximizing quantity is \( q_{ii} = (1 - \rho)/2 \). If \( s = d \), firms \( i \) and \( j \) compete in market \( i \) with costs \( \rho \) and \( c + \rho \), respectively. The Cournot-Nash equilibrium is such that \( q_{ii} = (1 + \rho - c)/3 \) and \( q_{ij} = (1 - 2c - \rho)/3 \) if \( \rho < 1 - 2c \), and \( q_{ii} = (1 - \rho)/2 \).
3.3 Overlap non-contractible and effort unobservable

We now turn to the case where the level of care exercised by the patent holder when drafting the field-of-use clauses is unobservable. What consequence does this have for the optimal contract? Notice first that the patent holder has no incentive to exert effort unless his royalty income (as opposed to the fixed fees which are paid upfront, before the realization of overlap) depends positively on \( e \). Thus, provision of effort is incompatible with the bunching scheme that is optimal under observability. It is also incompatible with a linear (constant per-unit) royalty. Of course, the licensees will anticipate this and, if offered such a scheme, will be willing to pay only their expected profit given \( e \) to obtain a license. Second, recall that implementability requires that monopoly output does not exceed duopoly output (Lemma 3). Hence, a scheme that induces effort must be such that output in the case of overlap is strictly greater than in the no-overlap case. Third, the royalty payment associated with the no-overlap quantity must exceed the payment associated with the overlap quantity. We can thus guess that the relevant incentive constraint will be the one for the no-overlap “type,” who may be tempted to mimic the overlap “type.”

The optimization problem is

\[
\max_{e;F; (q(m), r(m)); (q(d), r(d))} F + e r(m) + (1 - e) r(d) - \psi(e)/2
\]

subject to

\[
F \leq e^* \left[ \Pi_m(q(m)) - r(m) \right] + (1 - e^*) \left[ \Pi_d(q(d), q(d)) - r(d) \right] \tag{21}
\]

\[
\Pi_m(q(m)) - r(m) \geq \Pi_m(q(d)) - r(d) \tag{22}
\]

\[
\Pi_d(q(d), q(d)) - r(d) \geq \Pi_d(q(m), q(d)) - r(m) \tag{23}
\]

\[
\Pi_d(q(d), q(m)) - r(d) \geq \Pi_d(q(m), q(m)) - r(m), \tag{24}
\]

where \( e^* \) is the equilibrium level of effort (rationally anticipated by the licensees). Unlike in the previous section, the menu of outputs and royalty payments now has to accomplish two things: organize production efficiently, and provide incentives to the licensor to exert effort. While the problem of nonresponsiveness remains, it may now sometimes be optimal to induce separation, precisely in order to incentivize the licensor. To do so, \( q(d) \) must be raised above \( q(m) \), which is inefficient. Thus, the parties may decide to sacrifice some efficiency in exchange for higher effort. This can only be optimal, however, when the cost of effort is not too large,

\[q_{ij} = 0 \text{ if } \rho \geq 1 - 2c.\] Setting \( \rho = 1 - 2c \) is sufficient to shut down firm \( j \) in market \( i \). Thus, further increases in \( \rho \) can never be optimal as they would only distort firm \( i \)'s output further away from the optimal level. Summing \( q_{ii} \) and \( q_{ij} \) yields \( q^*(d) = (2 - 2p - c)/3 \).

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at least for small values of e (or more precisely, when it does not increase too rapidly with e). This idea is formalized in Assumption 3, which provides a sufficient condition on the cost of effort that guarantees the existence of a separating contract.

The alternative to a separating contract is a pooling contract. Pooling means that no effort beyond e can be sustained, so that the optimal solution becomes a contract with a quantity restriction \( \bar{q} \) chosen to maximize expected profits given e. The following proposition characterizes the optimal separating contract and gives sufficient conditions for this contract to dominate the optimal pooling contract.

Proposition 3. Suppose overlap is non-contractible, effort is unobservable, and Assumptions 2 and 3 hold. Then, there exists \( \bar{e} > 0 \) and \( q^u(d) > q^*_m \) such that, for all \( c \leq \bar{e} \), the optimal contract is \( (e^u, F^u, (q^u(m), r^u(m)), (q^u(d), r^u(d))) \) with the following properties:

(i) \( e^u \) is defined by \( \psi'(e^u) = 2[\pi'_m - \Pi_m(q^u(d))] \);

(ii) \( F^u = e^u[\pi'_m - r^u(m)] + (1 - e^u)[\Pi_d(q^u(d)) - r^u(d)] \);

(iii) \( q^u(m) = q^*_m \) and \( q^u(d) \) satisfies

\[
\frac{\partial \varphi(q^*_m, q^u(d))}{\partial q(d)} \left[ \pi'_m - \Pi_d(q^u(d)) - \psi'\left(\varphi(q^*_m, q^u(d))\right) \frac{1}{2} \right] = \left[ \varphi(q^*_m, q^u(d)) - 1 \right] \Pi'_d(q^u(d)),
\]

where \( \varphi(q(m), q(d)) \equiv (\psi')^{-1}[2(\Pi_m(q(m)) - \Pi_m(q(d)))] \).

(iv) \( (r^u(m), r^u(d)) \) satisfy \( r^u(m) - r^u(d) = \pi'_m - \Pi_m(q^u(d)) \).

This can be implemented with a royalty scheme \( R \) which takes the form of a step function, that is,

\[
R(q) = \begin{cases} 
   r^u(m) & \text{for } q < q^u(d) \\
   r^u(d) + \rho(q - q^u(d)) & \text{for } q \geq q^u(d),
\end{cases}
\]

where \( \rho \geq \partial \Pi_d(q^u(d), q^u(d))/\partial q_i \).

Proof: See Appendix A.

Proposition 3 shows that, under some assumptions on \( \psi \) and \( c \), when effort is unobservable, the optimal royalty scheme is no longer trivial and features royalties that are decreasing with output over some range: we have \( q^u(m) < q^u(d) \) and \( R(q^u(m)) > R(q^u(d)) \). There are other interesting observations. We have a variant of the well-known “no distortion at the top” result: in the absence of overlap, the contract provides for the efficient output, \( q^*_m \). This is natural since a separating contract aims at incentivizing the licensor. The binding incentive
constraint (22) means that effort is increasing in the difference between the no-overlap profits at \( q(m) \) and \( q(d) \). This difference is best maximized by setting \( q(m) \) at the profit-maximizing level.

There is a tradeoff between providing incentives to the licensor and producing at the efficient scale. In fact, while raising \( q(d) \) relaxes the incentive constraint and thereby induces greater effort, it also moves duopoly output away from the efficient level, \( q^*_d \). The intuition for why this can be optimal is the following. If \( \psi \) increases sufficiently slowly at low values of \( e \) (in the sense of the conditions on second and third-order derivatives stated in Assumption 3), a small difference between \( q(d) \) and \( q(m) \) translates into a large increase in effort (we are in the flat part of the cost curve). Moving \( q(d) \) upwards starting from \( q^*_m \) quickly increases the probability of monopoly, outweighing the decline in duopoly profits and the higher cost of effort. As \( q(d) \) continues to increase, \( e \) enters the steeper part of the cost curve, and eventually the negative effect on \( \Pi_d \) and effort costs comes to outweigh the positive effect on effort, so that the licensor’s payoff peaks at some quantity – namely, \( q^u(d) \).

The resulting payoff needs to be compared to the optimal pooling contract. Proposition 3 says that the separating contract can dominate for small values of \( c \) but is silent about larger ones. The payoff from the separating contract is difficult to compute, so the proof relies on a lower bound, reached at \( q(d) = q^*_m \), and then uses marginal deviations of \( q(d) \) from \( q^*_m \). If \( c \) is small, \( q^*_d \) is close to \( q^*_m \), and we know from Proposition 2 that the optimal pooling contract calls for a quantity between the two. Since, by definition, \( \hat{\Pi}'_d = 0 \) at \( q^*_d \), the effect of small deviations is second order, so \( \hat{\Pi}_d(q^*_m) \) is close to \( \pi^*_d \). The difference between the optimal pooling contract and the separating contract will then mainly be driven by the difference in effort. Provided \( \psi \) is sufficiently flat, a small increase in \( q(d) \) generates enough effort for separation to dominate pooling. We should note that the condition on \( c \) is a sufficient but by no means a necessary condition. Separation may well be superior to pooling even if \( c \) is large.

There are some degrees of freedom in the choice of \( r^u(m) \) and \( r^u(d) \). What is important is the difference between them. The fixed fee can again be used to extract the remaining (expected) surplus from the licensees.

4 Discussion

In this section we discuss the robustness of the results to departures from our basic modeling assumptions.
The nature of competition

Our setup with Cournot competition and linear demand has been widely used in the economic analysis of licensing. In the current context, the assumption that firms compete in quantities is particularly important. When firms instead compete à la Bertrand and their only strategic variable is price, \( P \) can use a simple royalty scheme such as a unit royalty, set the royalty rate such that the duopoly quantity is exactly equal to the optimal monopoly quantity \( (q^*_m) \), and choose \( e = 0 \), thereby achieving the joint profit maximizing outcome. There is no inefficiency since, under price competition with asymmetric costs, the high-cost firm’s output is zero. In that extreme case there is no reason for the use of FOUR.

A related point concerns product differentiation. We have used a two-market setting where the licensor’s effort in drafting the field-of-use clauses determines the probability of overlap. If there is overlap, licensees compete in each of the two markets. An alternative approach would consist in a one-market setting with horizontally differentiated products where the licensor’s effort determines how close to each other in the product space the licensees are allowed to locate. Greater effort would translate into a greater distance between the products, relaxing competition.\(^{24}\) Although intuitively appealing, the problem with such an approach is to come up with a model that is as rich as the two-market setting while at the same time remaining tractable. Consider for example a Hotelling model with transportation costs that are not too convex, so that the licensees’ equilibrium location choices absent FOUR would be in the interior of the line (see Economides, 1986). Like in the current model, the licensor’s effort would then increase joint surplus by alleviating a prisoner’s dilemma among the licensees. As long as the market is covered, however, equilibrium quantities in the Hotelling model are the same regardless of the licensor’s effort, and both quantities and profits are independent of cost (and thus also independent of royalties). Therefore, the Hotelling model is ill-suited for investigating how the design of royalties affects the licensor’s incentives to exert effort in defining fields of use, which is the focus of this paper.

Contributions by the licensees to the definition of fields of use and renegotiation

We have assumed that the task of drafting the license contract, including the definition of fields of use, is solely the responsibility of the licensor. This assumption can be justified by the fact that it is the licensor who developed the technology to be licensed. He may thus have private information regarding the attributes that are likely to be crucial for avoiding overlap, putting him in a better position to draft the field-of-use clauses than the licensees.

\(^{24}\) Such an approach would be in the spirit of Klemperer (1990).
But even if the licensees are in a position to contribute to the definition of fields of use, the licensor may well be the only one to have the incentives to do so. Each licensee firm would like to have a broad license itself while it would like the other licensee to have a narrow license. \(L_1\), say, wants to have a license that allows her to enter field of use 2 and wants \(L_2\) to have a license that is too restrictive to enter field 1. Therefore, \(L_1\) has no incentive to narrow down the definition of its own field of use. Moreover, as contracts are bilateral, \(L_1\) has no influence on \(L_2\)’s license contract.

A similar argument can be made with respect to renegotiation. Upon learning that there is overlap, the parties could gain collectively by readjusting the license contracts. However, since the contracts are bilateral agreements between licensor and licensee, there is little scope for realizing these potential gains. None of the licensees will find it in her interest to renegotiate her own contract. Another obstacle to renegotiation is that, even if the parties were able to negotiate a multilateral agreement, renegotiating after overlap has occurred would necessarily delay the actual market introduction of products. If we introduce a discount factor into the model, licensees might not be prepared to wait, especially if eliminating overlap is expected to require several rounds of negotiation (each time the contract terms are modified the parties need to check for overlap). Therefore, the effort invested in the initial contract would still be of central importance.

**Sequential product introduction**

In the model, licensees introduce their products simultaneously at date \(t = 2\). In practice, some further development is usually necessary before the products incorporating the licensor’s technology are marketable. It is likely that one of the licensees will achieve marketability before the other. How robust are the results to one of the licensees having a first-mover advantage?

This essentially changes competition in the case of overlap from a Cournot to a Stackelberg game. Note that for the symmetric treatment of licensees we have used throughout the paper to continue to make sense, it must not be known which licensee will achieve marketability first. Each licensee must be equally likely to be the first mover in the product market. We now show that if licensees are ex ante symmetric in this sense, our qualitative results are unchanged.

Suppose that, after date \(t = 1\) (where licensees learn the realization of overlap), there is an additional stage where nature draws which of the licensees moves first, and that both licensees have equal probability of being the first mover. The monopoly case is obviously
unaffected by who moves first. In the duopoly case, let us assume without loss of generality that firm 1 moves first and chooses quantities \( q_{11} \) and \( q_{12} \) subject to \( q_{11} + q_{12} = q_1(d) \). Firm 2 moves second and best responds to firm 1’s quantities subject to \( q_{22} + q_{21} = q_2(d) \); hence, assuming that both firms produce in equilibrium, firm 2’s output is still determined by (4), yielding
\[
q_{22}(q_{11}) = \frac{q_{11} + q_2(d) - (q_1(d) - c)/2}{2}; \quad q_{21}(q_{11}) = q_2(d) - q_{22}(q_{11}).
\]
Firm 1 now takes into account that its output affects firm 2’s output and solves
\[
\max_{q_{11}} [1 - q_{11} - q_{21}(q_{11})]q_{11} + [1 - q_{22}(q_{11}) - (q_1(d) - q_{11}) - c](q_1(d) - q_{11}).
\]
The equilibrium quantities are
\[
q_{11} = \frac{q_1(d) + 3c/2}{2} \tag{25} \\
q_{12} = \frac{q_1(d) - 3c/2}{2} \tag{26} \\
q_{22} = \frac{q_2(d) + 5c/4}{2} \tag{27} \\
q_{21} = \frac{q_2(d) - 5c/4}{2}, \tag{28}
\]
from which we deduce the licensees’ profits,
\[
\Pi_1^d(q_1(d), q_2(d)) = [1 - (q_1(d) + q_2(d))/2 - c/8](q_1(d) + 3c/2)/2 + [1 - (q_1(d) + q_2(d))/2 - 7c/8](q_1(d) - 3c/2)/2 \tag{29} \\
\Pi_2^d(q_2(d), q_1(d)) = [1 - (q_1(d) + q_2(d))/2 + c/8](q_2(d) + 5c/4)/2 + [1 - (q_1(d) + q_2(d))/2 - 9c/8](q_2(d) - 5c/4)/2. \tag{30}
\]
Summing both licensees’ profits, we observe that once again total industry profit depends only on the total output \( q_1(d) + q_2(d) \), and not on how it is shared between the licensees. We can thus restrict attention to symmetric schemes, such that \( q_1(d) = q_2(d) = q(d) \). It is straightforward to show that the joint profit maximizing quantity is
\[
q^{**}_d = \frac{1 - c/2}{2},
\]
which is the same as under Cournot competition: \( q^{**}_d = q^*_d \). Average industry profit (which is also each licensee’s expected profit from an ex ante perspective) is
\[
\bar{\pi}_d^{**} = \frac{2}{3}\sum_{i=1}^2 \Pi_i^d(q_i^{**}, q^{**}_d)/2 = \frac{1}{4} \left[ 1 - c + \frac{47}{16}c^2 \right],
\]

which is greater than under Cournot competition: \( \pi_d^{**} > \pi_d^* \).

Let us now check whether there are any changes to the set of implementable allocations. The incentive compatibility constraints are

\[
\Pi_m(q(m)) - r(m) \geq \Pi_m(q(d)) - r(d) \tag{33}
\]

\[
\Pi_2^1(q(d), q(d)) - r(d) \geq \Pi_2^1(q(m), q(d)) - r(m) \tag{34}
\]

\[
\Pi_2^2(q(d), q(d)) - r(d) \geq \Pi_2^2(q(m), q(d)) - r(m). \tag{35}
\]

The two constraints (34) and (35) can be collapsed into one because

\[
\Pi_2^1(q(d), q(d)) - \Pi_2^1(q(m), q(d)) = (q(d) - q(m))[1 - q(d) - (q(m) + c)/2] = \Pi_2^2(q(d), q(d)) - \Pi_2^2(q(m), q(d)).
\]

Adding up (33) and (34), we obtain, as in the case of simultaneous product introduction (see the proof of Lemma 3),

\[
(q(d) - q(m))(q(m) - c) \geq 0,
\]

implying that to be feasible, allocations must satisfy the monotonicity condition \( q(m) \leq q(d) \). Therefore, the essence of our analysis remains intact even if product introduction is sequential rather than simultaneous. The optimal level of effort is lower than under simultaneous product introduction because industry profit in the case of overlap is higher (and the gain from avoiding overlap thus lower). Qualitatively, however, the results remain the same.

**Observability of sales revenue rather than output**

We have assumed throughout the analysis that the licensor can observe each licensee’s total output but not prices. Alternatively, the licensor may be unable to observe either output or prices separately, yet may be able to observe the total revenue (or sales) of each licensee, \( pq \). Royalty payments would have to condition on revenue rather than output. As we now show, a simple percentage-of-sales royalty may then be able to provide incentives for effort provision. This is interesting because, as we have discussed, when output rather than revenue is observable, a simple unit royalty cannot induce any effort on the part of the licensor.

Suppose effort is unobservable and overlap non-contractible. Suppose moreover that the licensor can only observe the licensees’ sales and assume that the licensor uses a percentage-of-sales royalty. Letting \( \sigma_s(\rho) \) denote equilibrium sales in state \( s \) given a royalty rate \( \rho \), the licensor’s effort is then determined by

\[
\rho[\sigma_m(\rho) - \sigma_d(\rho)] = \psi'(e)/2.
\]
To compute $\sigma_s(\rho)$, consider the licensees’ problem. We can no longer normalize $c_{ii}$ to zero without loss of generality. Assume thus that $c_{ii} = c_L > 0$ and $c_{ij} = c_H > c_L$ for all $i, j \neq i$. If $s = m$, each licensee solves

$$\max_q (1 - \rho)(1 - q)q - c_Lq,$$

yielding $q_m(\rho) = \frac{1 - c_L/(1 - \rho)}{2}$. If $s = d$, licensee $i$ solves

$$\max_{q_{ii}, q_{ij}} (1 - \rho)[(1 - q_{ii} - q_{ji})q_{ii} + (1 - q_{ij} - q_{jj})q_{jj}] - c_Lq_{ii} - c_Hq_{ij}.$$

In equilibrium, we have, for $i = 1, 2$ and $j \neq i$,

$$q_{ii}(\rho) = \frac{1 + (c_H - 2c_L)/(1 - \rho)}{3}$$

$$q_{ij}(\rho) = \frac{1 + (c_L - 2c_H)/(1 - \rho)}{3}.$$

Computations show that the equilibrium sales of each licensee are given by

$$\sigma_m(\rho) = \frac{1}{4} \left( 1 - \frac{c_L^2}{(1 - \rho)^2} \right)$$

$$\sigma_d(\rho) = \frac{(1 - \rho + c_H + c_L)(2(1 - \rho) - (c_H + c_L))}{9(1 - \rho)^2}.$$

Sales are greater under monopoly than under duopoly, $\sigma_m(\rho) \geq \sigma_d(\rho)$, if and only if

$$(1 - \rho - 2c_H)^2 - c_L[4(1 - \rho - 2c_H) + 5c_L] \geq 0.$$

Assuming $1 - \rho - 2c_H \neq 0$, this condition is always satisfied if $c_L = 0$ and thus, by continuity, also if $c_L$ is positive but small. We conclude that there exist circumstances in which a percentage-of-sales royalty gives the licensor incentives to exert effort even though a unit royalty fails to do so. The intuition is the following. Under both a unit royalty and a percentage-of-sales royalty, total duopoly output is greater than monopoly output for any royalty rate. With a unit royalty, output is all that matters for the royalty payment received by the licensor. By contrast, a percentage-of-sales royalty also depends on the price at which the product is sold, and price and output move in opposite directions. Under certain conditions on costs and demand, the product of price and output is higher under monopoly than under duopoly.

An implication of this result is the following. Suppose that prior to the contracting stage, there is a pre-stage in which the licensor can invest in a monitoring technology. The

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25 If $c_{ii} = 0$, a percentage-of-sales royalty is non-distortive in the monopoly case, while for any $c_{ii} > 0$ it is.
monitoring technology can be configured to track either output or sales. The analysis above suggests that the licensor may prefer to invest in tracking sales rather than output. Unlike a unit royalty, a percentage-of-sales royalty can induce effort, and it may be easier to implement than the non-linear output-dependent royalty in Proposition 3.

5 Conclusion

We have studied how field-of-use restrictions and the risk of overlap between fields affect the design of license contracts. Our setting is that of an outside licensor whose technology has applications in two different fields of use, in each of which a different firm produces most efficiently. The licensor needs to exert effort in drafting the contract to clearly separate the fields of use. If he fails to do so, the fields overlap and each licensee can produce in both fields.

We have examined the optimal contract in three different environments. When overlap is contractible, the contract imposes an output restriction on licensees in the event of overlap, thus softening competition. It gives incentives to the licensor by means of a reward that the licensees pay in the absence of overlap. When overlap is non-contractible but effort observable, the optimal contract involves pooling: licensees are induced to produce the same output regardless of whether there is overlap. When overlap is non-contractible and effort unobservable, the license contract must organize production and at the same time provide incentives to the licensor. Under some conditions, we have shown that the optimal contract involves royalties that are decreasing with output. Thus, our model offers a rationale for the prevalence of such royalty schemes in practice (Taylor and Silberston, 1973; Kamien, 1992).

Two additional testable implications of the results obtained in the case where overlap is non-contractible and effort is unobservable are the following. First, if the licensor measures only production and not the quantities actually sold, and there is free disposal, the monopolist licensee will produce the duopoly quantity to pay lower royalties, and then sell only the (lower) monopoly quantity. Thus, the above royalty scheme is not incentive compatible, implying that effort above $e$ is not sustainable in equilibrium. Since this reduces the attractiveness of using FOUR in the first place, especially when $e$ is low, we might expect them to be used less when sales are hard to measure. Second, the extent to which effort is observable can be interpreted as a measure of the information gap between licensor and licensees, and/or the complexity of the technology. Along these lines, the model predicts that as the technology becomes more complex, license contracts with FOUR should make greater use of royalties.
To conclude, we briefly discuss some implications of our results for antitrust authorities. A direct implication of the model is that quantity restrictions and non-linear royalty schemes should not be considered per se violations of the antitrust laws when used in conjunction with FOUR. In the framework we examine, they lead to improvements in production efficiency. One qualification that needs to be made in this respect is the possibility that FOUR may be misused to restrain competition between firms that would otherwise be horizontal competitors. If the technology licensed represents only a minor improvement, competition is harmed by the agreement without any offsetting benefits in terms of higher quality. A simple rule that antitrust authorities could use to determine whether an agreement is likely to be welfare-enhancing is to check whether the licensees continue to sell the original product that does not incorporate the improved technology. In this way, if the improvement is minor, the improved product will not sell if its price greatly exceeds the price of the original product, as they will be close substitutes. And since the original product is supplied competitively, welfare is not likely to be harmed by the agreement.

Appendix A  Proof of Proposition 3

We know from Lemma 3 that the incentive compatibility constraints (22) and (23) imply the monotonicity condition $q(d) \geq q(m)$. By Proposition 2, a pooling allocation such that $q(d) = q(m)$ would maximize production efficiency. Constraint (22), however, means that if $q(d) = q(m)$, then $r(m) \leq r(d)$. Because equilibrium effort is determined by

$$r(m) - r(d) = \psi'(e)/2,$$

(A.1)

no effort beyond $e$ can be sustained if $q(d) = q(m)$.

The licensor would like to raise $r(m) - r(d)$ to increase equilibrium effort, but is constrained in his ability to do so by (22), which can be rewritten as

$$r(m) - r(d) \leq \Pi_m(q(m)) - \Pi_m(q(d)).$$

Constraint (23) can be rewritten as

$$r(m) - r(d) \geq \Pi_d(q(m), q(d)) - \Pi_d(q(d), q(d)).$$

This constraint puts a lower bound on $r(m) - r(d)$. Because the only reason to raise $q(d)$ above $q(m)$ is to raise $r(d) - r(m)$, we conclude that (22) must be the binding constraint.

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26 From an antitrust perspective, usage restrictions are generally considered benign (see, e.g., the Antitrust Guidelines for the Licensing of Intellectual Property issued by the US Department of Justice and the Federal Trade Commission in 1995, and Gilbert and Shapiro (1997)).
while (23) will be slack. From the proof of Lemma 3, we know moreover that (23), together with the monotonicity condition, implies the uniqueness constraint (24).

Combining (22) and (A.1), we have

\[ e^* = (\psi')^{-1} \left[ 2(\Pi_m(q(m)) - \Pi_m(q(d))) \right] \equiv \varphi(q(m), q(d)). \]

Using (21) and the fact that expectations concerning \( e \) must be correct in equilibrium, the licensor’s problem is to maximize with respect to \( q(m) \) and \( q(d) \)

\[ \varphi(q(m), q(d))\Pi_m(q(m)) + [1 - \varphi(q(m), q(d))]\hat{\Pi}_d(q(d)) - \psi(\varphi(q(m), q(d))) / 2. \]  

(A.2)

Optimizing with respect to \( q(m) \) yields

\[ \frac{\partial \varphi}{\partial q(m)} \left[ \Pi_m(q(m)) - \hat{\Pi}_d(q(d)) - \frac{\psi'(e^*)}{2} \right] + e^* \Pi'_m(q(m)) = 0 \]  

(A.3)

By the inverse function theorem, \( \partial \varphi / \partial q(m) = 2\Pi'_m(q(m)) / \psi''(e^*) \). Thus, the solution to equation (A.3) is \( q(m) = q^*_m \).

It follows from the monotonicity condition that \( q(d) \geq q^*_m \). We now show that, given the assumptions stated in the proposition, the objective function (A.2) is strictly increasing in \( q(d) \) at \( (q^*_m, q^*_m) \) and strictly decreasing as \( q(d) \) grows large, implying that there exists an interior solution \( q^u(d) > q^*_m \). Differentiating (A.2) with respect to \( q(d) \) yields

\[ \frac{\partial \varphi}{\partial q(d)} \left[ \Pi_m(q(m)) - \hat{\Pi}_d(q(d)) - \frac{\psi'(e^*)}{2} \right] + (1 - e^*)\hat{\Pi}'_d(q(d)). \]  

(A.4)

Evaluating this expression at \( q(m) = q^*_m \), we have

\[ \frac{\partial \varphi(q^*_m, q(d))}{\partial q(d)} \left[ \pi^*_m - \hat{\Pi}_d(q(d)) - \frac{\psi'(\varphi(q^*_m, q(d)))}{2} \right] + [1 - \varphi(q^*_m, q(d))]\hat{\Pi}'_d(q(d)). \]  

(A.5)

What we need to show is that the limit superior of this expression as \( q(d) \) tends to \( q^*_m \) is strictly positive. We have \( \partial \varphi / \partial q(d) = -2\Pi'_m(q(d)) / \psi''(e^*) \). As \( q(d) \) tends to \( q^*_m \), both numerator and denominator tend to zero since \( \varphi(q^*_m, q^*_m) = e \) and, by Assumption 3, \( \psi''(e) = 0 \). Applying l’Hôpital’s rule yields

\[ \lim_{q(d) \to q^*_m \atop q(d) > q^*_m} \frac{\partial \varphi(q^*_m, q(d))}{\partial q(d)} = \lim_{q(d) \to q^*_m \atop q(d) > q^*_m} \frac{-2\Pi''_m(q(d))}{\psi''(e^*)}. \]

Using the fact that \( \Pi''_m = -2 \) and rearranging, we obtain

\[ \lim_{q(d) \to q^*_m \atop q(d) > q^*_m} \frac{\partial \varphi(q^*_m, q(d))}{\partial q(d)} = \frac{2}{\sqrt{\psi''(e)}}. \]
Further computations yield $\hat{\Pi}_d(q^*_m) = (1 - c + 2c^2)/4$, $\psi'(\varphi(q^*_m, q^*_m)) = 0$ and $\hat{\Pi}'(q^*_m) = -c/2$, so the limit superior of the expression in (A.5) is positive if

$$\frac{2}{\sqrt{\psi''(q)}} \frac{c(1 - 2c)}{4} - (1 - e) \frac{c}{2} > 0.$$ 

By Assumption 3, $\psi'''(e) \leq [c(1 - 2c)]^2$. The worst case for our required result is equality, in which case we obtain

$$\frac{1}{2} - (1 - e) \frac{c}{2} > 0,$$  

which is true for any $c$. Thus, (A.2) initially increases as one raises $q(d)$ above $q^*_m$. By the Inada conditions, effort becomes infinitely costly as it approaches 1, so the objective function eventually decreases with $q(d)$ as $q(d)$ grows large. By the intermediate value theorem, there exists at least one local maximum such that $q(d) > q^*_m$. Any such solution must satisfy the first-order condition

$$\frac{\partial \varphi(q^*_m, q(d))}{\partial q(d)} \left[ \pi^*_m - \hat{\Pi}_d(q(d)) - \frac{\psi'(\varphi(q^*_m, q(d)))}{2} \right] + \left[ 1 - \varphi(q^*_m, q(d)) \right] \hat{\Pi}'_d(q(d)) = 0.$$ 

(A.7)

Denote by $q^u(d)$ the best interior solution, which represents the optimal separating contract. We now show that, if $c$ is not too large, the licensor’s payoff from a separating contract with $q^u(d)$ is always greater than the payoff from the optimal pooling contract. By Proposition 2, the optimal pooling contract involves an effort of $\varepsilon$ and a quantity $\bar{q} = \left[ 1 - (1 - \varepsilon)c/2 \right]/2$. These entail an expected payoff of $[4 - 4c(1 - \varepsilon) + c^2(9 - 10\varepsilon + \varepsilon^2)]/16$. Evaluating (A.2) at $(q^*_m, q^*_m)$ yields $[\varepsilon + (1 - \varepsilon)(1 - c + 2c^2)]/4$. The difference between the two is $[c(1 - \varepsilon)]^2/16$. Clearly, as $c \to 0$, the difference also tends to zero. The derivative of the payoff from a separating contract at $q(d) = q^*_m$ is given by equation (A.6). Its limit is never lower than $1/2$ as $c \to 0$, and it is independent of, or decreasing in, $c$. It follows that there exists $\bar{c} > 0$ such that, for all $c \leq \bar{c}$, the optimal separating contract $(q^*_m, q^u(d))$ yields a higher payoff than the optimal pooling contract.

As for the last claim, the optimal royalty scheme needs to satisfy $R'(q^*_m) = 0$ in order for the licensees to choose $q^*_m$ in the case of exclusivity; in the case of overlap, from the equilibrium condition of the Cournot game played by the licensees when facing a royalty scheme $R$, for $q(d)$ to be the optimal choice of quantity for firm $i$ given that firm $j$ chooses $q(d)$, we must have

$$\frac{\partial \Pi_d(q(d), q(d))}{\partial q_i} \leq R'(q(d)).$$

That is, given $q_j = q(d)$, the marginal royalty must exceed the marginal profit from increasing
Since the proposed $R(q)$ is linear in $q$ for $q \geq q^u(d)$, while $\Pi_d$ is concave, it is sufficient that the condition holds at $q^u(d)$. ■

Appendix B An exclusive license covering both fields of use

Throughout the paper, we take it as given that the licensor prefers licensing two firms to licensing a single firm. We now consider the option of giving one firm an exclusive license for both fields of use. In that case, the firm is a monopolist in both fields but produces with an inefficient technology in one of them. The corresponding level of profits is

$$\pi_b = \pi_m^* + \max_q (1 - q - c)q = \frac{1}{4} + \frac{(1 - c)^2}{4} = \frac{2(1 - c) + c^2}{4}.$$  

Let us compare $\pi_b$ to the level of profit the licensor can obtain by giving a non-exclusive license to both firms. As shown in the text, a non-exclusive license should restrict output to $q_d^*$, i.e., the level that maximizes industry profits. If $c < 2/5$, joint profits are

$$2\pi_d^* = \frac{2(1 - c) + 9c^2/2}{4}.$$  

Clearly, $2\pi_d^* \geq \pi_b$ for any $c < 2/5$. If $2/5 \leq c < 1/2$, joint duopoly profits are $2c(1 - c)$, which is greater than $\pi_b$ if and only if

$$\frac{5 - \sqrt{7}}{9} \leq c \leq \frac{5 + \sqrt{7}}{9}.$$  

This interval strictly includes $[2/5, 1/2)$. Thus, non-exclusively licensing two firms always dominates licensing a single firm supplying both markets. The intuition is that, while the exclusive licensee and the duopoly licensees produce the same total quantity (namely, $1 - c/2$), the duopoly produces a larger share of it with the efficient technology.

Proposition 2 has shown that the licensor prefers licensing with field-of-use restrictions to non-exclusive licensing when $c < 2/5$. Proposition 3 similarly derives conditions under which the licensor prefers a separating contract to a pooling contract, which, in turn, is preferred to non-exclusive licensing. The above results imply that FOUR will also be preferred over an exclusive license to a single firm. For the case $c \geq 2/5$, FOUR do at least as well as non-exclusive licenses. This is because at $q = c$, $\hat{\Pi}_d = \Pi_m$, so a contract with FOUR designed such that $\tilde{q} = c$ and $\epsilon = \epsilon$ yields the same payoff as non-exclusive licenses. If the licensor chooses $\tilde{q} > c$, it must be that this procurest him a higher payoff. Thus, FOUR always dominate an exclusive license for both fields.
Appendix C  A direct revelation mechanism that implements first best

Assume that only the licensees learn the state of the world (i.e., the realization of overlap) at date \( t = 1 \) of the game. Consider the following direct revelation mechanism. Both licensees are asked to report the state of the world \( s \). The allocation that results depends on both licensees’ reports and is determined by the following table where each cell contains a quadruplet \((q_i, r_i), (q_j, r_j)\), that is, quantity-transfer pairs for licensees \( L_i \) (row player) and \( L_j \) (column player).

<table>
<thead>
<tr>
<th>( m )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (q_m^<em>, \pi_m^</em> - \pi_d^<em>), (q_m^</em>, \pi_m^* - \pi_d^*) )</td>
<td>( (\hat{q}, \hat{r}), (\hat{q}, \hat{r}) )</td>
</tr>
<tr>
<td>( (\hat{q}, \hat{r}), (\hat{q}, \hat{r}) )</td>
<td>( (\hat{q}, 0), (\hat{q}, 0) )</td>
</tr>
</tbody>
</table>

Table C.1: Allocations resulting from the message game

Can we find values for \((\hat{q}, \hat{r}, \tilde{q}, \tilde{r})\) such that truthtelling is the unique Nash equilibrium of the game? For a truthful equilibrium to exist, it must be the case that

\[
\Pi_m(q_m^*) - (\pi_m^* - \pi_d^*) \geq \Pi_m(\tilde{q}) - \tilde{r} \tag{C.1}
\]

\[
\Pi_d(q_d^*, q_d^*) \geq \Pi_d(\hat{q}, \hat{q}) - \hat{r}. \tag{C.2}
\]

Moreover, in order not to get a nontruthful equilibrium, the following constraints must be satisfied:

\[
\Pi_m(q_d^*) < \Pi_m(\hat{q}) - \hat{r} \tag{C.3}
\]

\[
\Pi_d(q_m^*, q_m^*) < \Pi_d(\hat{q}, \hat{q}) - \hat{r}. \tag{C.4}
\]

Let us set \( \hat{q} = q_d^* + \varepsilon \), where \( \varepsilon \) is arbitrarily small, and \( \hat{r} = 0 \). This satisfies (C.3), and also ensures that (C.2) is satisfied for any \( \tilde{q} \geq q_d^* \). Adding up the remaining two constraints (C.1) and (C.4), and noting that \( \Pi_m(q_m^*) = \pi_m^* \) and \( \Pi_d(q_d^*, q_d^*) = \pi_d^* \), we obtain the following condition:

\[
\Pi_m(\tilde{q}) - \pi_d^* \leq \tilde{r} < \Pi_d(\hat{q}, q_d^*) - \Pi_d(q_m^*, q_m^*). \]

For such an \( \tilde{r} \) to exist, it must be the case that, for some \( \tilde{q} \geq q_d^* \),

\[
\Pi_m(\tilde{q}) - \Pi_d(\tilde{q}, q_d^*) < \pi_d^* - \Pi_d(q_m^*, q_m^*). \]

The right-hand side of this expression is necessarily strictly positive. Thus, it suffices to find a \( \tilde{q} \) such that

\[
\Pi_m(\tilde{q}) = \Pi_d(\tilde{q}, q_d^*). \tag{C.5}
\]
We show that such a $\tilde{q}$ exists for the case $c < 2/5$, i.e., where both licensees are involved in production. Solving (C.5), we obtain

$$\tilde{q} = \frac{1}{4} + \frac{3c + \sqrt{A}}{8},$$

where $A = (2 + 11c)(2 + 3c)$. Since $A$ is positive, $\tilde{q}$ exists.

References


